Solving Parallel Problems

Unconventional Formula Suitable for Quick Mental Calculations

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MOST of us who are concerned with radio in its many and varied aspects have occasion to use Ohm's law, and early in our acquaintance with this law we discovered that when using resistances in series the value was easy to find. We just added their respective values. When we came to putting resistances in parallel we found that it was a different matter. There was a formula :—

$$R = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}} \text{ etc.}$$

Some of us do not carry around slide-rules for working out reciprocals and the rule of thumb routine is apt to be tedious. There was an alternative formula for two resistances in parallel :—

$$R = \frac{R_1 \times R_2}{R_1 + R_2}$$

This is the accepted formula at present in general use, and unless the values of R_1 and R_2 are fairly simple figures it still requires either the slide-rule or pencil and paper.

Now when we parallel two resistances, we know at once that the combined value will be lower than that of either single resistance. Let us start from the most elementary values and observe their relationships. To find the combined value R when we parallel R_1 of 1 Ω with R_2 of 2 Ω; using the conventional formula we find that the answer is 2/3 Ω, which is 1/3 Ω lower in value than R_1. So far there is nothing significant except that we note that a new figure, 3, appears in the answer which is related to 2 and 1. Let R_1 be 1 Ω and R_2 be 10 Ω. Then R is 0.909 Ω. R_1 was 1/10th of the value of R_2, but R is not 1/10th lower in value than R_1 but 1/11th, i.e., R_1 minus (R_1 divided by 10 plus 1). Now add some noughts to the figures and let us see if this relationship still holds good. Let R_1 be 1,000 Ω and R_2 be 10,000 Ω. Then R is 909 Ω, still 1/11th lower than R_1.

Take some different figures: let R_1 be 56 Ω, and R_2 be 280 Ω (five times greater), then R is 46.6 Ω, which is 1/6th lower than R_1. What is happening is that the combined value is following a relationship which is in proportion to the "ratio" of the resistance values, with the distinction that there is always figure 1 added; so all we need to do is to look at the values of R_1 and R_2, mentally assess the ratio of R_2/R_1, and add 1. Divide R_1 by this figure and subtract the result from R_1 to get the answer for R.

For example: find the parallel value R when R_1 is 33 Ω and R_2 is 66 Ω. The ratio R_2/R_1 is 66/33 = 2/1. Add 1, which gives the divisor 3. Then R = 33 − 33/3 = 33 − 11 = 22 Ω. This method can be stated in the formula :—

$$R = R_1 - \frac{R_1}{\frac{R_2}{R_1} + 1}$$

which can be shown to be the equivalent of the usual formula.

Having shown that resistances in parallel follow a "law" related to the ratio of their values, a further simplification can be introduced, namely :—

$$R = \frac{R_2}{\frac{R_2}{R_1} + 1}$$

which, again, can be shown to be the equivalent of the usual formula.

We need only look on R_2/R_1 as a simple ratio to get sufficient accuracy for most practical requirements. Even where the experienced designer of radio circuitry wants accuracy with speed it will surely be agreed that this unconventional formula has advantages over the generally accepted one.

To take an example, let R_1 be 8,000 Ω and R_2 be 24,000 Ω. Then substituting values, the parallel value will be :—

$$R = \frac{24{,}000}{\frac{24{,}000}{8{,}000} + 1} = \frac{24{,}000}{3 + 1} = 6{,}000\,\Omega$$

Reversing the values of R_1 and R_2 :—

$$R = \frac{8{,}000}{\frac{8{,}000}{24{,}000} + 1} = \frac{8{,}000}{\frac{1}{3} + 1} = 8{,}000 \div \frac{4}{3} = 6{,}000\,\Omega$$

The formula applies equally to impedances whereever phase angle is not involved. Where Z is resistive :

$$Z = \frac{Z_2}{\frac{Z_2}{Z_1} + 1}$$

which is a useful formula for the quick calculation of parallel loads. Similarly for parallel inductances :—

$$L = \frac{L_2}{\frac{L_2}{L_1} + 1}$$

Parallel capacitances are simply added together as are series resistances, but we can adapt the formula for capacitances in series :—

$$C = \frac{C_2}{\frac{C_2}{C_1} + 1}$$

Simple examples have, of course, been chosen for convenience in demonstrating the general principle. The formula is easy to memorize, and speed in using it will soon come with practice. Finally, here is a useful application of the "ratio" idea which needs no formula to express it. We have a resistance (or a load) of 55 Ω and we want to bring it down to 50 Ω. Let R be 50 Ω and R_1 be 55 Ω and R_2 be the parallel value required. Visualize R_1 split into its two parts (R_1 minus R) and R. Then (R_1 − R) is to R, as R_1 is to R_2. **That is, 5 : 50 as 55 : R_2. Almost at a glance R_2 is 550 Ω.**